Topic Study Group 41

Research and Development on Textbooks and Resources for Learning and Teaching Mathematics

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ABSTRACT This chapter gives an overview of the themes that grounded the work of Topic Study Group 41 at ICME-14, and an account of the congress sessions, in which the contributions were reported and discussed. The chapter authors were the members of the TSG-41 organizing team, chaired by S. Rezat and co-chaired by J. Visnovska. We highlight the directions for the continuation of the work of this TSG in the areas of (a) production of detailed accounts and justifications of the principles used in resource design, (b) resource design and evaluation for the opportunities to teach, (c) exploration of the supports the students require to navigate the creation of their own learning trajectories, and (d) development of analytical tools for determining the specific ways in which teaching resources could contribute to teacher support needs within broader systemic improvement efforts.

Keywords: Mathematics textbooks, Curriculum resources, Digital resources, Teaching resources, Instructional design.

1. Themes and Description of TSG-41

The efforts of Topic Study Group 41 (from here on TSG-41) focused on explorations of the issues related to the contents, design, development, use, and implementation of print and digital resources for teaching and learning of mathematics. The resources included extend beyond the print and digital school textbooks, and include teacher manuals, professional development materials, student learning and assessment materials, and a variety of online resources.

Directly linking to, and building on, the earlier work of TSG-38 at ICME-13 (Fan et al., 2017; Fan et al., 2018) the aim of TSG-41 at ICME-14 was to bring to the foreground and examine various theoretical and methodological approaches used to design, analyze, and empirically study mathematics learning and teaching resources and their use in diverse geographic regions and contexts. The pre-congress call for

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contributions for TSG-41 highlighted three broad leading themes of (1) role and effects, (2) design and development, and (3) use and implementation of mathematics resources. The aim was to attend to resources across their creation, use, and effects, with attention paid to the changes introduced by the emergence of digital resources. Each of the three themes in the call included illustrative questions.

Theme 1: The role and effects of print and digital textbooks and other resources in mathematics classrooms.

- What teaching and learning resources are available in mathematics classrooms in different countries? What role do they play in mathematics teaching, learning, and assessment? What are the current effects of the use and implementation of these resources on student learning?
- How is the role that the resources play affected by the digitalization of information and communication and the growing availability of digital and online resources? What are the effects of modern ICT (particularly internet) on students' use of learning resources?
- How does the availability and use of digital resources affect student behavior, learning, and relationships to the subject of mathematics?

Theme 2: The design and development of print and digital mathematics textbooks and other resources.

- What are the theoretical foundations that guide the aspects of development and design of mathematics textbooks and other resources, such as the selection and progression of tasks, development of student competencies, considerations of and supports for envisioned teacher learning or change of practice, features of interactive elements and feedback in digital resources?
- What do we know about designing resources for supporting specific pedagogical intentions for mathematical learning (e.g., project-based, inquiry-based or problem-based learning) and for supporting mathematical learning in environments with blended agendas (e.g., integrated, STEM, multiliteracies)?
- What are the key differences in features and contents between print and digital resources that result from the affordances of digitalization? How do we conceptualize interactions between resource designers and users? Specifically, what is the role of teachers and students in developing textbooks and other teaching and learning materials?

Theme 3: The use and implementation of print and digital mathematics textbooks and other resources and related interactions among resources, teachers, and students.

- What are the influences on the use and implementation of textbooks and other resources? How are teachers supported in their interaction with and the implementation of textbooks and other resources?
- How do teachers adopt and adapt new resources in their professional work?
- How do teachers' individual resources interact with collective resources, and how could we model such relationships?
- What are the consequences of the use of particular resources for the teaching of mathematics, and for teacher knowledge and professional development?
- What resources do students use for learning mathematics and how do they use them?
- How do students, as well as their teachers, interact through resources?

2. Organization of TSG-41 Sessions

Being the group that deals with research on textbooks, TSG-41 received comparatively high number of submissions. After the review process, 40 contributions were accepted, of which six were presented as long oral presentations, 20 as short oral presentations, eight as posters, and six author teams withdrew their submissions due to the ICME-14 date changes and transition to a hybrid congress mode in 2021.

2.1. Oral presentations

In addition to submitted contributions, TSG-41 engaged three top scholars as invited speakers and discussants, tasked with providing direction for the work of the group.

The ICME-14 congress assigned each TSG three sessions, two 90 min long, and one 120 min long. We included one invited talk in each session, and gave all long oral presenters time for an individual presentation.

Aiming for fruitful experiences online as well as on site, the organizing team sought creative ways of fostering discussions and collaborations, in which authors of all accepted short and long oral presentations could actively take part. Therefore, in the lead up to the congress, thematic small groups of paper authors have met online to discuss their respective work and co-create a pre-recorded 5 min video update on the issues and questions that stood out in their combined contributions.

Subsequently, the congress TSG blocks were shared between three 20-minute discussant contributions, six 10-minute long-paper presentations, and seven 5-minute pre-recorded reports from small group pre-congress debates. Moreover, we arranged $10 + 15 + 20$ minutes of discussions (scheduled respectively at the end of each presentation block), as well as opening and closing remarks from the organizing team. The list of oral presentations is given in Tab. 1, while the list of all accepted paper contributions that reflects the pre-congress collaborative work in small groups is given in Tab. 2 (each long oral presentation appears in both tables with the same numbering).

Tab. 1. List of Invited Talks and Long Oral Presentations

Paper and author(s)
Invited discussant contributions
[1] Textbooks as teacher support for engaging students in active knowledge organization. ***Susanne Prediger*** (Germany).
[2] Digital mathematics curriculum resources: Towards design principles of educative materials for students and teachers. ***Birgit Pepin*** (The Netherlands).
[3] Instructional materials as tools for instructional improvement. ***Paul Cobb*** (USA).
Long oral presentations
[4] Identifying educative features in scripted mathematics lesson plans. ***Moneoang Leshota*** (South Africa).
[5] Learning to design resources for teachers. ***Jana Visnovska*** (Australia), *José Luis Cortina* (Colombia), *and Pamela Val* (South Africa).
[6] Elements of a theory of textbook design. ***Sebastian Rezat*** (Germany).
[7] Didactic considerations regarding the iterative development design of dynamic digital tools. ***Anatoli Kouropatov***, *Regina Ovodenko, Michal Fraenkel, Maureen Hoch* (Israel).
[8] Teaching and learning with dynamic textbooks: studying student uses at scale. ***Vilma Mesa*** *and Saba Gerami* (USA).
[9] Investigating the use of mathematics textbooks by students in Shanghai and England: a comparative study. ***Yi Wang*** (China) *and Lianghuo Fan* (UK/China).

Tab. 2. List of papers presented in Groups

Paper and author(s)
Group 1
[10] An analysis of data and probability tasks in US and Chinese elementary mathematics textbooks. ***Xiang Gao*** (China).
[11] Constructing a textbook analysis framework of statistics and probability areas in elementary math. ***Shiqi Lu*** *and Wenbin Xu* (China).
[12] Learning to design resources for teachers. ***Jana Visnovska*** (Australia), *José Luis Cortina* (Colombia), *and Pamela Val* (South Africa).
[12] The effect of the curricula on textbooks for the teaching of probability and statistics. ***Gergely Balazs Wintsche*** (Hungary).
[13] Mathematics Education according to the textbook: opportunities to learn investigated. ***Marc van Zanten*** (The Netherlands) *and Marja van den Heuvel-Panhuizen* (Norway).
Group 2
[7] Didactic considerations regarding the iterative development design of dynamic digital tools. ***Anatoli Kouropatov****, Regina Ovodenko, Michal Fraenkel, Maureen Hoch* (Israel).
[4] Identifying educative features in scripted mathematics lesson plans. ***Moneoang Leshota*** (South Africa).
[14] A comparative study of bidirectional connections in U.S.A. and Chinese high school mathematics textbook problems. ***Shuhui Li*** (USA/China).
[15] Translations of function representation in different textbooks. ***Yang Shen*** *and Jiansheng Bao* (China).
Group 3
[16] A comparative study of problem solving in Chinese and U.S.A. primary mathematics textbook. ***Suijun Jia*** (China).
[17] A comparative analysis of tasks contexts in mathematics textbooks in China and Singapore. ***Yao Li*** *and Lianchun Dong* (China).
[18] A comparative study of mathematical inquiry activities in textbooks in China and Singapore. ***Hongwei Ran*** *and Lianchun Dong* (China).
[6] Elements of a theory of textbook design. ***Sebastian Rezat*** (Germany).
Group 4
[8] Teaching and learning with dynamic textbooks: studying student uses at scale. ***Vilma Mesa*** *and Saba Gerami* (USA).
[19] The elements of textbooks that Indonesian mathematics teachers use as they adopt student-centered instructional approach. ***Dewi Rahimah*** *and Jana Visnovska* (Australia).
[9] Investigating the use of mathematics textbooks by students in Shanghai and England: a comparative study. ***Yi Wang*** *and Lianghuo Fan* (UK/China).
Group 5
[20] Sesamath resources and collective work from mathematical laboratory to classes in Arabic environment. ***Karima Sayah*** (Algeria).
[21] Promoting the teaching and learning of mathematics through visualising connections in post-16 resources. ***Dominic R. Oakes*** *and Sofya Lyakhova* (UK).
[22] Comparing naming systems used by Chinese and Ukrainian teachers: exploring teachers' resource system. ***Maryna Rafalska*** (France), *Chongyang Wang* (China), *and Luc Trouche* (France).
Group 6
[23] Toward systematic support for preservice teachers' learning of productive resource use. ***Ok-Kyeong Kim*** (USA).
[24] Analysing teachers' individual and collective resources through the lens of their digital resources. ***Katiane de Moraes Rocha*** (Brazil).
[25] Student understanding of textbook visual representations of natural and fractional numbers: a collaborative international research. ***Everaldo Silveira*** (Brazil) *and Arthur B. Powell* (USA).
[26] How expert mathematics teacher design curriculum based on textbook use: a case study in Beijing. ***Guorui Yan*** (Hong Kong SAR, China).
Group 7
[27] Long-term use of a digital mathematics textbook with integrated digital tools: investigating the influence on students' achievement and self-efficacy. ***Maxim Brnic*** (Germany).
[28] The relationship between mathematical examples in Malawian grade 1 primary school mathematics teachers' guide and the goals of outcome-based education. ***Lisnet Mwadzaangati*** (Malawi).
[29] Career mathways: a teaching & learning intervention to show the relevance of mathematics in careers. ***Niamh O'Meara****, Olivia Fitzmaurice, and Patrick Johnson* (Ireland).

2.2. Contributed Posters

Posters in this TSG are listed in Tab. 3.

Tab. 3. List of posters

Poster and author(s)	
[30]	The extent of creative reasoning opportunities and aspects of cognition demand in textbooks in Nepal: a case of high school mathematics textbook. *Deepak Basyal and* ***Mohan Thapa*** (Nepal).
[31]	This is the way I use textbooks and other resources for design mathematics lessons: A case of teaching the area of circle. ***Ya Cheng*** (China).
[32]	Educative curriculum materials: Teachers' continuous training in the step by step of the materials designing process. ***Pauli Diniz*** (Mozambique).
[33]	About textbooks on mathematical logic and theory of algorithms for prospective mathematics teachers. ***Vladimir I. Igoshin*** (Russia).
[34]	The presentation of core knowledge acquisition process in junior middle school mathematics textbooks. ***Tianzhuo Jiang*** *and Shuwen Li* (China).
[35]	The presentation of linear function in Chinese school mathematics textbooks. ***Na Li*** (China).
[36]	A comparative study on fractions in primary school's mathematics textbooks of China and the United States. ***Fulin Liu****, Yiming Cao, and Dengfeng Liang* (China).
[37]	A comparative study of "figures and geometry" in junior middle school mathematics textbook by PEP edition and Kangxuan edition. ***Yihan Wang****, Meiyue Jin, and Jiadi Zhang* (China).

3. Contributions to the Themes

3.1. Theme 1

The three contributions to TSG-41 in group 7 were related to Theme 1 (the role and effects of print and digital textbooks and other resources in mathematics classrooms), and particularly related to the question of how the availability and use of digital resources affect student behavior, learning, and relationships to the subject of mathematics. Brnic[27] reported on the long-term effects of a digital textbook on secondary students' achievement and their self-efficacy, where, so far, no significant effects were found for either construct. O'Meara et al.[29] investigated how Irish textbooks include examples, which show the relevance of mathematics in careers. Finding the lack of such examples in the textbooks, the authors created other materials that show the relevance of mathematics and tested them in a pilot study. Mwadzaangati[28] analyzed Malawian primary level mathematics textbooks in order to identify how the examples in these textbooks contributed to the goals of outcome-oriented education. The starting point of all three papers was that textbooks still appear to be the dominant resource used in classrooms across the world. While the paper by Mwadzaangati[28] focused on the contribution of the textbooks to achieve the goals of the official curriculum, the two papers by Brnic[27] and O'Meara et al.[29] focused on alternatives to the traditional textbook and explored the affordances of other resources for enhancing the learning of mathematics.

3.2. Theme 2

Papers in Groups 1, 2, and 3 contributed to Theme 2 (the design and development of print and digital mathematics textbooks and other resources). Rezat[6] raised the need for the theoretical foundations capable of guiding the development and design of mathematics textbooks and other resources. He offered a preliminary attempt at systematizing the empirically evaluated design principles and features of mathematics textbooks, and at forming a structural theory of textbook and curriculum material design.

Some contributions within this theme could be regarded as being related to the design of particular opportunities to learn that mathematics textbooks and other resources come to embody (or capture, or represent). Design research provides a methodological framework that aligns theoretical considerations in the design process with empirical evaluations of the design aims. Two papers exemplified these design processes, while using different methodological approaches. Kouropatov et al.[7] described the iterative design process of three digital resources related to transformations of functions. The paper demonstrated how mathematical and didactical considerations must go hand in hand with empirical evaluations in the design process to develop resources that have the desired learning effects. Similarly, Lisnani and Sariyasa[6] described the design and testing of a digital resource, in which they leveraged comics for learning integers.

A number of contributions to TSG-41 did not bring accounts of design principles or the design process but analyzed and evaluated the opportunities to learn as they were provided in mathematics textbooks (both historical and those currently in use). These papers used comparative analysis to highlight specific design features of textbooks from different countries, different historical periods, or different pedagogical approaches. Using a post-design perspective, such analyses may unveil the design principles and decisions as manifested in the resource from an a posteriori perspective, thus deepening our understanding of resource design. In particular, the contributed studies analyzed comparatively textbooks in China and the U.S.A. (Gao[10], Jia[16], Li[14]), China and Singapore (Li and Dong[17], Ran and Dong[18]); textbooks with different approaches in China (Shen and Bao[15]) and in the Netherlands (van Zanten and van den Heuvel-Panhuizen[13]), and textbooks from different historical periods in Hungary (Wintsche[12]). Finally, Lu and Xu[11] focused on the construction of a framework for analyses of this kind.

Alongside the majority of studies that explored designs from perspective of learning, researchers and designers remain aware that textbooks rarely teach students directly. As a result, the notion of textbook designs, which would not only provide opportunities to learn for students, but also *opportunities to teach* for teachers, surfaced

[6] *Teaching and learning integers through ICT-based SI UNYIL comics*. Short paper. The paper was not assigned a discussion group, but the authors were able to attend TSG-41.

within the discussions related to this theme. Visnovska et al.[5] reported on the processes of developing resources that would be in service of teachers (as opposed to teachers having to be in service of the resource). They called for the design of the features that would contribute to teachers' re-claiming the agency over both the meanings and decisions associated with mathematics teaching. Similarly, Leshota[4] analyzed educative features in scripted mathematics lesson plans from South Africa and explored the opportunities that the materials presented for the teachers' learning and teaching.

3.3. Theme 3

Papers in groups 4, 5, and 6 contributed to Theme 3 (The use and implementation of print and digital mathematics textbooks and other resources and related interactions among resources, teachers, and students). Within this theme, several authors foregrounded the supports for teachers' use and implementation of textbooks and other resources. Kim[23] combined five components, that prior research identified as key for teachers' productive use of resources, into a framework and explored its usefulness with the U.S.A. pre-service teachers who learned how to use resources. Oakes and Lyakhova[21] used interviews, surveys and questionnaires to identify the extent to which postgraduate level resources were seen to support making mathematical connections and whether the users thought that this was improving their teaching and learning. Rafalska et al.[22] explored the linguistic and cultural supports, aiming to develop a deeper understanding of cultural differences in resource systems of teachers from China and Ukraine. Rahimah and Visnovska[19] analyzed an Indonesian textbook from the perspective of the kinds of supports it provided, or failed to provide, for teachers' implementation of a student-centered teaching approach.

Within the scope of investigations of teachers' resource systems and documentational trajectories, Sayah[20] explored how one Algerian teacher adopted and adapted the French Sésamath resources in their professional work, and how these resources fostered the collective work of this teacher. Rocha[24], in turn, analyzed the interaction of the individual and collective resources in the case of one French mathematics teacher, and Similarly, Yan[26] documented how one Chinese expert teacher used the textbook as a resource in her curricular design.

Additional papers explored students' use of curriculum resources and what sense the students were making of the presented content. Wang and Fan[9] surveyed Chinese and English secondary students' use of mathematics textbooks, highlighting striking differences in the perceived purposes for and uses of the textbooks. Mesa and Gerami[8] tracked U.S.A. university students' viewing of a digital textbook in real time and collected students' written narratives about their textbook use and reported on the methodological challenges faced in analyzing this type of data. Silveira and Powell[25] reported on the methodology developed for explorations of elementary students'

understanding of visual representations of natural numbers and fractions in select Brazilian and U.S.A. textbooks.

4. Conclusion and Outlook

A number of ideas discussed within TSG-41 appear to be worth carrying forward in the research of future TSG participants.

Many contributions to TSG-41 had shown that textbook and resource designs differ, at times profoundly, for instance in different countries. It is certain that these differences contribute to how teachers can teach and how and what students get to learn and accomplish in mathematics classrooms. However, it is rarely clear which of the documented differences are to be attributed to specific design assumptions, principles, and processes, as those are rarely explicit, and often remain hidden from our views. In other words, for those of us whose research involves resource design, there is a need for more explicit sharing of the accounts and justifications of the principles that guide our current designs, as well as of processes that guide our theoretical and empirical evaluations of these principles in different cultural and institutional contexts.

The latter requirement calls for deeper empirical substantiation of the effects of textbook and resource design. While understanding differences in textbook and resource design as well as particular values and intentions and ways in which these are used to drive designs is important, it is also necessary, and still not common, to investigate whether or to what extent these values and intentions get realized when the textbooks are used in classrooms. Prediger's discussant contribution offered an inspirational example from the KOSIMA project of how research can be structured to intentionally contribute to the production of such accounts. She illustrated how ongoing cycles of empirical evaluations of both the task design and teacher supports were essential in the production of mathematics education resources that positively contributed to students' learning. This links to the need for the pursuit of analytical notions such as resources for teaching, teachers' resources, and opportunities to teach, which were brought up in several paper contributions and small group discussions during TSG-41.

In addition to the design work that is conducted by resource designers and by teachers, Pepin's discussant contribution drew attention of the role of students as designers of their own learning trajectories and thus as co-designers of curriculum. Accordingly, connectivity, in terms of making the connections among mathematical ideas within mathematical curricula explicit to the students, becomes a critical feature of curriculum design, as it plays a role in supporting students in navigating the resources needed for their own curriculum trajectories.

Finally, Cobb's discussant contribution focused on the role that textbooks and other resources (i.e., instructional materials) play within the broader concerted efforts at instructional improvement, especially when attempts are made to orchestrate these at the systems level. Drawing on the MIST project data, Cobb illustrated that the

quality of instructional resources made a considerable difference in student achievement scores, even when teachers used the cognitively demanding tasks in somewhat proceduralized ways. This insight is important given that textbooks which are not composed of well-sequenced mathematical tasks of high cognitive demand are still widely available to schools and teachers. It should be concerning that textbooks of this kind directly contribute to portraying students in ways, which do not represent their capacity for mathematical learning, and they do so at scale. Cobb hastened to stress that in order for teachers to thrive while using high-quality teaching resources, teachers must be adequately supported in this use. How textbooks and other resources could be designed to better support teachers' work is worth of concerted research efforts.

To conclude, we would like to extend an invitation to researchers of textbooks and other curricular and instructional resources to join in the events specifically dedicated to this work, including the 4th International Conference on Mathematics Textbook Research and Development (ICMT 4) in Beijing (China) in October 2022, and the next iteration of this TSG (i.e., TSG3.12) at ICME-15 in Sydney (Australia), July 2024.

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